Spiked Eigenvalues of High Dimensional Separable Sample Covariance Matrices

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November 19, 2019
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The model

\[ y_{it} = \ell_{i1} f_{1t} + \ell_{i2} f_{2t} + \varepsilon_{it} = \ell^*_i f_t + \varepsilon_{it}, \quad i = 1, 2, \ldots, n; \quad t = 1, 2, \ldots, T, \quad (1) \]

where \( f_t = (f_{1t}, f_{2t})^* \) are two common factors, \( \ell_i = (\ell_{i1}, \ell_{i2})^* \) are the corresponding factor loadings, and \( \varepsilon_{it} \) is the error component, in which the symbol "*" denotes the conventional conjugate transpose.
Scenario: No true common factors

Under this case, the factor loadings are generated as $\ell_i = (0, 0)^*$. When the original data follow AR(1) model ($\gamma = 0.2$), Figures 1 and 2 provide all eigenvalues of the sample covariance matrix as $(T, n) = (20, 40)$ and $(T, n) = (40, 20)$, respectively. There are no spiked eigenvalues in view of these graphs, which correctly reflect the fact that there are no common factors in the original data.
Figure: $T = 20, n = 40, \gamma = 0.2$
Figure: $T = 40, n = 20, \gamma = 0.2$
Figure: $T = 20, n = 40, \gamma = 1$
Figure: $T = 40, n = 20, \gamma = 1$
Scenario: No true common factors

However, as the data observations are nonstationary ($\gamma = 1$), Figures 3 and 4 show that there is one spiked eigenvalue from the sample covariance matrix, while the true number of common factors is 0. This example demonstrates that PCA may not be informative accurately on high dimensional data with dependent sample observations.
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Consider an $n$-dimensional random vector $\mathbf{y}$ with observations $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T$. Pool all observations together into a $T \times n$ matrix $\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T)^*$. The data matrix $\mathbf{Y}$ has the structure

$$\mathbf{Y} = \Gamma \mathbf{X} \Omega^{1/2},$$

where $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_n) = (x_{ij})_{(T+L) \times n}$ is a $(T + L) \times n$ random matrix with i.i.d. elements; $\Sigma = \Gamma \Gamma^*$ and $\Omega$ are $T \times T$ and $n \times n$ deterministic non-negative definite matrices, respectively. Here $\Gamma$ is a $T \times (T + L)$ deterministic matrix.
Actually the matrix $\Gamma$ describes dependence among sample observations.

The matrix $\Omega$ measures cross-sectional dependence for $y$ under study.

Under this setting, the sample covariance matrix of $y$ can be expressed as $\Gamma X \Omega X^* \Gamma^*$. It is also called separable covariance matrix.
Largest spiked eigenvalues

- We are interested in the largest spiked eigenvalues of matrix $\Omega$, which describes the cross-sectional dependence.
- In the classical procedure of using PCA, spiked empirical eigenvalues from sample covariance matrix $\Gamma X \Omega X^* \Gamma^*$ are utilized to approximate those of the matrix $\Omega$.
- In this paper, we investigate the spiked empirical eigenvalues from an innovative view: how the the spiked eigenvalues of the matrix $\Sigma$ (due to the dependent sample) affect the spiked sample eigenvalues?
- To this end, we do not impose any spiked structures on the matrix $\Omega$. 
We assume spikiness of the matrix $\Sigma$ through the following decomposition. Let the spectral decomposition of $\Gamma$ be $V \Lambda^{1/2} U$, where $V$ and $U$ are $T \times T$ and $T \times (T + L)$ orthogonal matrices respectively ($VV^* = UU^* = I$), $\Lambda$ is a diagonal matrix composed by the descent ordered eigenvalues of $\Sigma = \Gamma \Gamma^*$. Moreover, we write $\Lambda = \begin{pmatrix} \Lambda_S & 0 \\ 0 & \Lambda_P \end{pmatrix}$, where $\Lambda_S = \text{diag}(\mu_1, \ldots, \mu_K)$, $\Lambda_P = \text{diag}(\mu_{K+1}, \ldots, \mu_T)$, and $\mu_1, \ldots, \mu_K$ are referred to the spiked eigenvalues that are significantly bigger than the rest. In addition, we write $U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$ and $\Sigma_2 = U_2^* \Lambda_P U_2$. 
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This section is to establish the asymptotic distribution of the largest spiked empirical eigenvalues. First, we make the following assumptions.

**Assumption (Moment Conditions)**

\[
\{x_{ij}: i = 1, \ldots, T + L, j = 1, \ldots, n\} \text{ are i.i.d random variables such that}
\]

\[E x_{ij} = 0, \quad E |\sqrt{n}x_{ij}|^2 = 1 \text{ and } E |\sqrt{n}x_{ij}|^4 = \gamma_4 < \infty.\]
Assumption 2

Assumption (Dependent Sample Structure)

\[ \alpha_L = \mu_K = \ldots = \mu_{K-n_L} < \alpha_{L-1} = \mu_{K-n_L+1} \ldots < \alpha_1 = \mu_{n_1} = \ldots = \mu_1, \]

where \( n_1, \ldots, n_L \) are finite. Moreover, there exists a small constant \( c > 0 \) such that \( \alpha_{i-1} - \alpha_i \geq c\alpha_i \) for \( i = 1, 2, \ldots, L \) and \( \mu_K - \mu_{K+1} \geq c\mu_K \).
Assumption 3

Assumption (Cross-sectional Structure)

The matrix $\Omega$ is nonnegative definite and its effective rank
\[ r^*(\Omega) = \frac{\text{tr}(\Omega)}{\|\Omega\|_2} \to \infty, \text{ where } \|\Omega\|_2 \text{ means the spectral norm.} \]
Assumption (Spiked Dependent Sample Structure)

The spiked eigenvalues of the population covariance matrix are much bigger than the rest of the eigenvalues. Precisely speaking, for $\forall \epsilon > 0$, there is $K_\epsilon$, independent of $n$ and $T$, such that when $n$ and $T$ are big enough,

$$\sum_{i=K_\epsilon}^{T} \frac{\mu_i}{\mu_K} < \frac{\epsilon}{2}.$$  \hspace{1cm} (3)
Remarks about Assumptions

Note that Assumptions 2 and 4 impose a spiked structure on \( \Sigma \) while Assumption 3 could endure either spiked or non-spiked structure on \( \Omega \). This is consistent with the aim of this paper to investigate the effect caused by dependent sample observations on the spiked sample eigenvalues.
Remarks about Assumptions

When $\mu_i = i^{-1-\sigma}$ and $\sigma > 0$ one can find that Assumption 4 holds. Moreover, Section 4.2 below shows that Assumption 4 holds in the unit root setting. In addition, define a near unit root model of the form:

$$y_{it} = \rho y_{i,t-1} + \sum_{h=0}^{L} b_h z_{i,t-h},$$

where $T(1 - \rho)$ is bounded as $T$ goes to infinity. It can also be verified that Assumption 4 holds in such models. Also, heterogeneous high-dimensional time series models can also be covered if the corresponding variances satisfy Assumption 4.
Denote the $i$-th largest sample eigenvalue of $\Gamma X \Omega X^* \Gamma^*$ by $\lambda_i$. Set $m_i = \sum_{j=1}^{i-1} n_j$, for all $i = 1, 2, \ldots, \mathcal{L}$. The following theorem establishes the asymptotic joint distribution of the largest spiked eigenvalues.
Suppose that Assumptions 1-4 hold,

\[
\frac{n}{\mu_i \sqrt{\text{tr}(\Omega^2)}} \left( \lambda_{m_i+1} - \mu_i \frac{\text{tr}\Omega}{n}, \lambda_{m_i+2} - \mu_i \frac{\text{tr}\Omega}{n}, ..., \lambda_{m_i+n_i} - \mu_i \frac{\text{tr}\Omega}{n} \right) \xrightarrow{d} \mathcal{R}_i,
\]

where \( \mathcal{R}_i \) are the eigenvalues of an \( n_i \times n_i \) matrix \( \mathcal{R}_i \) with the Gaussian elements, \( \mathbb{E} \mathcal{R}_i = 0 \), the covariance of the \((\mathcal{R}_i)_{k_1,l_1}\) and \((\mathcal{R}_i)_{k_2,l_2}\) is

\[
\lim_{n \to \infty} \frac{n^2}{\text{tr}(\Omega^2)} \times \text{Cov}(u_{m_i+k_1}^* X\Omega X^* u_{m_i+l_1}, u_{m_i+k_2}^* X\Omega X^* u_{m_i+l_2}).
\]

Here the limit of (6) is bounded.
Moreover, if $\frac{\mu_i}{\mu_j} \geq c > 1$, $\lambda_{m_i+f}$ and $\lambda_{m_j+g}$ are asymptotically independent, where $1 \leq f \leq n_i$ and $1 \leq g \leq n_j$. Particularly when $n_i = 1$ for all $i = 1, \ldots, K$ we have

$$
\begin{align*}
\frac{\lambda_1 - \mu_1 \frac{tr\Omega}{n}}{\mu_1 \sqrt{var(u_1^*X\Omega X^* u_1)}}, & \quad \frac{\lambda_2 - \mu_2 \frac{tr\Omega}{n}}{\mu_2 \sqrt{var(u_2^*X\Omega X^* u_2)}}, \ldots, \\
\frac{\lambda_K - \mu_K \frac{tr\Omega}{n}}{\mu_K \sqrt{var(u_K^*X\Omega X^* u_K)}}, & \quad \xrightarrow{d} N(0, I_K).
\end{align*}
$$
Remarks about the Theorem

Note that

$$\sqrt{\frac{tr(\Omega)}{\|\Omega\|_2}} = \frac{tr(\Omega)}{\sqrt{\|\Omega\|_2}tr(\Omega)} \leq \frac{tr(\Omega)}{\sqrt{tr(\Omega^2)}} \leq \frac{tr(\Omega)}{\|\Omega\|_2}. \quad (7)$$

From this and Assumption 3, we can find that the standard deviation of $\lambda_i$ has the smaller order than the mean of $\lambda_i$. So the sample eigenvalues $\{\lambda_i, i \leq K\}$ have the same order as $\mu_i \frac{tr\Omega}{n}$. 

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Remarks about the Theorem

It indicates that the sample eigenvalues $\lambda_1, \cdots, \lambda_K$ are spiked under this case no matter whether $\Omega$ has spiked eigenvalues or not. This phenomenon suggests that PCA may reflect inaccurate information of the cross-sectional structure $\Omega$ due to the dependent sample observations. This is in contrast to the results Baik and Silverstein (2006) for the independent sample observations which establish one to one correspondence between the sample spiked eigenvalues and the population spiked eigenvalues due to the cross-sectional structure $\Omega$. 
Two Propositions

The following two propositions further investigate the relations between the leading sample eigenvalues and the eigenvalues of $\Sigma$ due to the dependent sample observations.

Proposition

Under the conditions of Theorem 2, there exists a positive constant $c$ such that $\lim \inf_{T \to \infty} \frac{\mu_1}{\text{tr}(\Gamma\Gamma^*)} > c$ and

$$\lim_{n,T \to \infty} P \left( \frac{\lambda_1}{\text{tr}(\Gamma X\Omega X^*\Gamma^*)} > c \right) = 1.$$  \hfill (8)

Moreover, when $1 \leq i < K$,

$$\frac{\lambda_i}{\lambda_{i+1}} - \frac{\mu_i}{\mu_{i+1}} \to 0 \text{ in probability as } n, T \to \infty.$$  \hfill (9)
Let the conditions of Theorem 2 hold. For $1 \leq i < K - 1$, if
\[
\min \left\{ \frac{\mu_i}{\mu_i + 1}, \frac{\mu_i + 1}{\mu_i + 2} \right\} \geq c > 1,
\]
then
\[
\frac{\lambda_i - \lambda_{i+1}}{\mu_i - \mu_{i+1}} = \frac{\lambda_i - \lambda_{i+1}}{\mu_i + 1 - \mu_{i+1}}
\]
has the same asymptotic distribution as
\[
\frac{v_1 - v_2}{v_3 - v_2},
\]
where $\{v_i : 1 \leq i \leq 3\}$ are i.i.d standard normal random variables.
Remark about Propositions

Remark

Proposition 1 shows that the ratio of the neighboring spiked empirical eigenvalues approximate that of the spiked eigenvalues from the dependent sample structure. A central limit theorem is provided for the ratio statistic constructed from the spiked empirical eigenvalues in Proposition 2.
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Proposition 1 implies that the largest sample eigenvalue has the same order as the sum of all eigenvalues.

The largest eigenvalue is so large that the methods in Ahn and Horensten (2013) and Bai (2004) would both estimate the number of factors to be the one bigger than zero even though there is no factor in our model.

Similarly, the relation between $\lambda_i$ and $\lambda_{i+1}$ leads to that Onatski (2010) would estimate the number of factors to be the one bigger than zero even though there is no factor in our model. We examine this one by one below.
The method in Onatski (2010) is based on the difference between the \(i\)-th largest eigenvalue and the \(i+1\)-th one. In a few words, the idea of Onatski (2010) is that if there is no factor, for any \(i \geq 1\), the difference between the \(i\)-th largest eigenvalue and the \(i+1\)-th one should be very small.

Recalling (9), we can find that the difference between the \(i\)-th largest eigenvalue and the \(i+1\)th one in our model can be large when \(\frac{\mu_i}{\mu_{i+1}} > 1\).

In other words, the method in Onatski (2010) would get a non-zero estimate for the number of factors in our model when \(\frac{\mu_i}{\mu_{i+1}} > 1\).
The method in Ahn and Horensten (2013) is based on the ratio between two successive largest eigenvalues. It defines a mock eigenvalue \( \lambda_0 = \frac{\sum_{i=1}^{\min\{n,T\}} \lambda_i}{\ln(\min\{n,T\})}. \) Then the estimator is

\[
\tilde{k}_{ER} = \max_{0 \leq k \leq k_{max}} \frac{\lambda_k}{\lambda_{k+1}}.
\] (10)

Note that \( \lambda_0 \) has a smaller order than the trace of the sample covariance matrix. Then the method of Ahn and Horensten (2013) implies that if there is no factor, the largest eigenvalue should have a smaller order than the trace of the sample covariance matrix.

(8) shows that the largest eigenvalue in our model has the same order as the trace of the sample covariance matrix.

In other words, the method in Ahn and Horensten (2013) would get a non-zero estimate for the number of factors in our model.
The method in Bai (2004)

- The methods in Bai (2004) is based on penalty functions. Briefly speaking, the idea of Bai (2004) is that if there is no factor, the largest eigenvalue should be smaller than the penalty function. The criterion has the form:

\[
PC(k) = \min_{\Lambda} \frac{1}{nT} \sum_{i=1}^{n} \sum_{t=1}^{T} (X_{it} - \lambda_{i}^{k'} \hat{F}_{t}^{k})^2 + kg(n, T), \tag{11}
\]

where \(g(n, T)\) satisfies some properties in Bai (2004).

- However, we can find that all the penalty functions have a smaller order than the trace of the sample covariance matrix.

- Recalling (8), the methods in Bai (2004) would get a non-zero estimate for the number of factors in our model.
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The unit root model

\[ y_{it} = y_{i,t-1} + \sum_{h=0}^{L} b_h z_{i,t-h}, \]  
(12)

where \(1 \leq i \leq n\), \(1 \leq t \leq T\) and \(L\) can be finite or infinite. Here

\[ z_{it} = \sum_{s=1}^{n} \gamma_{is} x_{st}, \]  
(13)

where \(1 \leq i \leq n\) and \(1 - L \leq t \leq T\).
Let $Y = (y_{it})$ be an $n \times T$ matrix and $\bar{Y}$ be an $n \times T$ matrix with all entries of the $i$th row being $\frac{\sum_{t=1}^{T} y_{it}}{T}$. We next specify some conditions so that Theorem 2 can be applied to the sample covariance matrix: $(Y - \bar{Y})^*(Y - \bar{Y})$. 
Assumption 5

\{x_{it}: i = 1, ..., n, t = 1 - L, ..., T\} are independent random variables such that \(E x_{it} = 0\). \(E |\sqrt{n} x_{it}|^2 = 1\) and \(E |\sqrt{n} x_{it}|^4 = \gamma_4 < \infty\). Write \(X = (x_1, ..., x_n)\).
Assumption 6

Assumption (Cross-sectional Structure)

\[ \Omega = \Upsilon^* \Upsilon \] satisfies Assumption 3 with the \( n \times n \) matrix \( \Upsilon = (\Upsilon_{is}) \).
Assumption 7

Assumption (Serial Correlation)

The coefficients \( \{b_i\}_{i=0}^L \) in (12) satisfy \( \sum_{i=0}^L i|b_i| < \infty \) and \( \sum_{i=0}^L b_i \neq 0 \).
Write $H = I - \frac{11^*}{T}$, where the $T \times 1$ vector $1$ consists of all one. Let $\Gamma = HCW$ and $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_T$ be the ordered eigenvalues of $\Gamma \Gamma^*$. With a simple calculation, the sample covariance matrix can be expressed as

$$(Y - \bar{Y})^*(Y - \bar{Y}) = HY^*YH^* = \Gamma X \Omega X^* \Gamma^*. \quad (14)$$
Theorem of the unit root

Let Assumptions 5-7 hold. Denoting the $k$-th largest eigenvalue of (14) by $\lambda_k$, for any fixed $k$, we have

$$
\frac{n}{\sqrt{2tr(\Omega^2)}} \left( \frac{\lambda_1 - \mu_1 \frac{tr\Omega}{n}}{\mu_1}, \frac{\lambda_2 - \mu_2 \frac{tr\Omega}{n}}{\mu_2}, \ldots, \frac{\lambda_k - \mu_k \frac{tr\Omega}{n}}{\mu_k} \right) \xrightarrow{d} N(0, I_k). \tag{15}
$$

Propositions 1-2 hold as well for model (12).
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Two models

We below focus on a nonstationary factor model of the form:

\[ M1 : \ y_{it} = \ell_i^* f_t + u_{it}, \]  

(16)

where \( f_t \) is a \( r \)-dimensional (\( r \) is fixed) vector, \( u_{it} \) is a stationary term and \( \{ f_t \}_{t=1,\ldots, T} \) are independent of \( \{ u_{it} \}_{i=1,\ldots, n, t=1,\ldots, T} \).

We then recall the unit root model discussed in Theorem 3 as follows:

\[ M2 : \ y_{it} = y_{i,t-1} + \sum_{h=0}^{L} b_h z_{i,t-h} \]  

(17)

for \( 1 \leq i \leq n \) and \( 1 \leq t \leq T \), where \( L \) can be finite or infinite,

\[ z_{it} = \sum_{s=1}^{n} \gamma_{is} x_{st} \]  

and Assumptions 5-7 hold.
some remarks

Note that Model M1 is equivalent to the following form:

\[
M3 : \ y_{it} = y_{i,t-1} + \ell_i^*(f_t - f_{t-1}) + u_{it} - u_{i,t-1} \triangleq y_{i,t-1} + v_{it}, \quad (18)
\]

which seems to be similar to model M2. However M3 is different from M2 in two aspects.

- At first, if \( f_t - f_{t-1} \neq 0 \), \( \ell_i^*(f_t - f_{t-1}) \) could lead to a strong cross-sectional dependence (strong factor) such that Assumption 6 is violated.

- Furthermore, even if \( f_t - f_{t-1} = 0 \), then \( v_{it} = u_{it} - u_{i,t-1} \) doesn’t necessarily satisfy \( \sum_{i=0}^{L} b_i \neq 0 \) in Assumption 7.

- Here one should note that \( (\sum_{i=0}^{L} b_i)^2 \) contributes to the limit of the first few largest eigenvalues of the corresponding sample covariance matrix.
Differences between M1 and M2

As a result, the eigenvalues of these two models behavior differently.

- When the number of factors in M1 is $r$, there is a significant drop from the $r$-th largest eigenvalue of M1 to its $(r + 1)$-th largest eigenvalue.

- In contrast the ratio of the $i$-th largest eigenvalue of M2 to its $(i + 1)$-th largest eigenvalue is asymptotically equal to $(i + 1)^2/i^2$ so that there is no significant drop between them.
We below propose a new statistic for M1 and M2. Define

\[ \bar{\mu}_i = \frac{1}{2} \frac{1}{1 + \cos \left( \frac{(T-i)\pi}{T} \right)} \] (19)

and

\[ T_{uf} = \frac{\lambda_1 \frac{\bar{\mu}_2}{\bar{\mu}_1} - \lambda_2}{\lambda_2 - \lambda_3 \frac{\bar{\mu}_2}{\bar{\mu}_3}}. \] (20)
The new test statistics

**Proposition**

Under the conditions of Theorem 3, for any fixed $k$,

$$
\lim_{n,T \to \infty} P \left( \tilde{k}_{ER} = \max_{0 \leq i \leq k} \left\{ \frac{\lambda_i}{\lambda_{i+1}} \right\} = 1 \right) = 1. \tag{21}
$$

Furthermore, when

$$
\lim_{n,T \to \infty} \frac{\text{tr}(\Omega)}{T \sqrt{\text{tr}(\Omega^2)}} = 0, \tag{22}
$$

the statistic $T_{uf}$ has the same asymptotic distribution as $\frac{v_1 - v_2}{v_2 - v_3}$, where $\{v_i : 1 \leq i \leq 3\}$ are i.i.d standard normal variables.
The new test statistics

Remark

Note that \( \frac{\text{tr}(\Omega)}{\sqrt{\text{tr}(\Omega^2)}} \leq \sqrt{n} \). So when \( \frac{\sqrt{n}}{T} \rightarrow 0 \), (22) holds.

Equation (21) implies that using \( \tilde{k}_{ER} \) in Ahn and Horensten (2013) may mistakenly think a unit root model as a single factor model.
The new test statistics

However using the statistic $T_{uf}$ could distinguish between them because for single factor models (see Assumption 1 in Onatski (2010), Assumptions A and B in Bai (2004) and Assumption A in Ahn and Horensten (2013)),

$$\frac{\lambda_1}{\lambda_2} \to \infty \text{ in probability as } n, T \to \infty.$$  \hspace{1cm} (23)

This ensures the power of the statistic $T_{uf}$ specified below.

**Proposition**

In single factor models (under the assumptions of Onatski (2010), Bai (2004) or Ahn and Horensten (2013)), the following holds

$$T_{uf} \to \infty \text{ in probability as } n, T \to \infty.$$  \hspace{1cm} (24)
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We have more thoughts about distinguishing four kinds of panel data structures:

1. stationary and weak cross-sectional dependence;
2. stationary and strong cross-sectional dependence;
3. unit root and weak cross-sectional dependence;
4. unit root and strong cross-sectional dependence.

Here by strong cross-sectional dependence we mean that its effective rank
\[ r^*(\Omega) = \frac{tr(\Omega)}{\|\Omega\|_2} \to c > 0 \]
while weak cross-sectional dependence implies that its effective rank
\[ r^*(\Omega) \to +\infty. \]
Theorem 2.3 of Zhang, Pan and Gao (2018) shows that when the data belongs to the first kind, the largest eigenvalue of sample covariance matrix has smaller order than the trace.

On the other hand, the largest eigenvalues from three other types of data have the same order as the trace.

So we can distinguish the first type, stationary and weak cross-sectional dependence, from others.
For the remaining three cases, we consider using PCA for them.

- Since PCA is a linear combination of data on the cross section we believe it has the same time-dependence as the initial data.
- In other words, from the first PC we may tell the difference between the second type stationary structure and the remaining two nonstationary structures since there are plenty of methods available for the univariate variable.
- So we can distinguish the second case from two others.
- Finally, we can use $T_{uf}$ to distinguish the third case from the fourth case.
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At first we compute the critical value by simulating \( \frac{v_1 - v_2}{v_2 - v_3} \) where \( \{v_i : 1 \leq i \leq 3\} \) are i.i.d standard Gaussian random variables based on 500000 replications. The quantiles of \( \frac{v_1 - v_2}{v_2 - v_3} \) are reported in Table 1.

**Table:** The quantiles of \( \frac{v_1 - v_2}{v_2 - v_3} \) based on 500000 replications

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<tr>
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<th>2.5% quantile</th>
<th>5% quantile</th>
<th>95% quantile</th>
<th>97.5% quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-11.6549</td>
<td>-6.0392</td>
<td>4.9932</td>
<td>10.4598</td>
</tr>
</tbody>
</table>

Then we can use a two-tailed test with the critical values -11.6549 and 10.4598. We can also use one-side test with the critical values \( C_{5L} = -6.0392 \) or \( C_{95R} = 4.9932 \).
We consider the following setting:

\[ y_{it} = y_{i,t-1} + \psi z_{i,t-1} + z_{it}, \]

where \( \psi = 0.5 \) and \( \Omega = \begin{pmatrix} \Omega_{i,j} \end{pmatrix} = \begin{pmatrix} 0.3|i-j| \end{pmatrix} \). The estimated sizes for the test statistic \( T_{uf} \) based on 1000 replications, different critical values and different values of \( n \) and \( T \) are reported in Tables 1-3. Tables 1-3 show that \( T_{uf} \) has stable sizes with different critical values. We can choose -11.6549 and 10.4598 as the critical values of a two-tailed test.
Table: 1 The size results on $T_{uf}$ based on 1000 replications

<table>
<thead>
<tr>
<th>n</th>
<th>T</th>
<th>two-side</th>
<th>$C_{5L}$</th>
<th>$C_{95R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
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<td>0.069</td>
<td>0.064</td>
<td>0.052</td>
</tr>
<tr>
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<td>0.051</td>
</tr>
<tr>
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<td>0.055</td>
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<td>0.060</td>
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<td>0.054</td>
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<td>0.070</td>
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</tr>
<tr>
<td>40</td>
<td>60</td>
<td>0.055</td>
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<td>0.057</td>
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<tr>
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<td>80</td>
<td>0.052</td>
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<td>0.047</td>
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<tr>
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<td>0.054</td>
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<tr>
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<td>200</td>
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<td>0.059</td>
<td>0.046</td>
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</tbody>
</table>
Table: 2 The size results on $T_{uf}$ based on 1000 replications

<table>
<thead>
<tr>
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<th>$C_{5L}$</th>
<th>$C_{95R}$</th>
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<tbody>
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<tr>
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<td>0.047</td>
<td>0.050</td>
<td>0.047</td>
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<td>0.056</td>
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<tr>
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<tr>
<td>80</td>
<td>200</td>
<td>0.054</td>
<td>0.044</td>
<td>0.055</td>
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</table>
The size results on $T_{uf}$ based on 1000 replications

<table>
<thead>
<tr>
<th>n</th>
<th>T</th>
<th>two-side</th>
<th>$C_{5L}$</th>
<th>$C_{95R}$</th>
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<tbody>
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<td>0.042</td>
</tr>
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<td>60</td>
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<td>0.044</td>
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<tr>
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<tr>
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<td>200</td>
<td>0.066</td>
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<td>0.063</td>
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</table>
We also calculate the proportion of $\tilde{k}_{ER} = 1$ with different values of the prescribed upper bound $k$ in (21). Tables 4-6 show that (21) also works well, since the calculated proportion is approaching 1 as the dimension $n$ and $T$ both increase.
Table: 4 The proportion of $\tilde{k}_{ER} = 1$ based on 1000 replications

<table>
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<tr>
<th>n</th>
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<th>$k = 10$</th>
<th>$k = 15$</th>
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<tbody>
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<td>0.732</td>
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<tr>
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Table: 5 The proportion of $\tilde{k}_{ER} = 1$ based on 1000 replications

<table>
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<th>$k = 10$</th>
<th>$k = 15$</th>
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<td>0.920</td>
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<td>200</td>
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</table>
Table: 6 The proportion of $\tilde{k}_{ER} = 1$ based on 1000 replications

<table>
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<th>$k = 10$</th>
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<tbody>
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<td>0.950</td>
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<tr>
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<td>60</td>
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<td>0.931</td>
<td>0.931</td>
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<tr>
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<td>0.989</td>
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<td>0.982</td>
<td>0.982</td>
<td>0.982</td>
</tr>
</tbody>
</table>
Outline

1 Motivation
   - Misleading PCA on Simulated Data
2 High Dimensional Separable Covariance Model
3 Asymptotic Performance of Largest Eigenvalues
4 Inference on High Dimensional Time Series
   - Implementing Factor Analysis on Our Model
   - Unit Root Models Satisfying Assumption 4
   - A New Test for Unit Root against Factor Model
   - More Thoughts about Panel Data Structures
5 Simulations
   - The Simulation about Proposition 3
   - The Simulation about Proposition 4
6 Reference
Now we consider the single factor model:

$$y_{it} = l_i f_t + \sqrt{\theta} e_{it}.$$  \hfill (25)

We use the same error term $e_{it}$ as in Ahn and Horensten (2013):

$$e_{it} = \sqrt{\frac{1 - \rho^2}{1 + 2J\beta}} \tilde{e}_{it}$$

$$\tilde{e}_{it} = \rho \tilde{e}_{i,t-1} + e_{it} + \sum_{h = \max(i - J, 1)}^{i - 1} \beta \epsilon_{ht} + \sum_{i + 1}^{h = \min(i + J, n)} \beta \epsilon_{ht},$$  \hfill (26)

where $\epsilon_{it}$ and $l_i$ are all drawn independently from $N(0, 1)$. We also use the most complicated setting of Ahn and Horensten (2013) which has both serially and cross-sectionally correlated errors: $\rho = 0.5$, $\beta = 0.2$ and $J = \max(10, n/20)$. 

Since $\rho < 1$, we can find that the error term is stationary. If $f_t$ is also stationary, $y_{it}$ is stationary. Then it will be very different from the unit root model and there are too many methods to test stationary and unit root. So we focus on the case where $f_t$ is nonstationary. We set $f_t = f_{t-1} + \tilde{f}_t$, where $\tilde{f}_t$ is drawn independently from $N(0, 1)$.

Then the power of $T_{uf}$ based on 1000 replications, different $\theta$, the critical values of the two-sided test and different values of $n$ and $T$, are reported in Tables 7-9. The power results given in Tables demonstrate that the proposed test works well numerically.
The power

Table: 7 The power results on $T_{uf}$ based on 1000 replications and two-sided test

<table>
<thead>
<tr>
<th>n</th>
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<th>$\theta = 1$</th>
<th>$\theta = 1/3$</th>
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</table>
### Table: 8 The power results on $T_{uf}$ based on 1000 replications and two-sided test

<table>
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</table>
Table: The power results on $T_{uf}$ based on 1000 replications and two-sided test

<table>
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Motivation
- Misleading PCA on Simulated Data

High Dimensional Separable Covariance Model

Asymptotic Performance of Largest Eigenvalues

Inference on High Dimensional Time Series
- Implementing Factor Analysis on Our Model
- Unit Root Models Satisfying Assumption 4
- A New Test for Unit Root against Factor Model
- More Thoughts about Panel Data Structures

Simulations
- The Simulation about Proposition 3
- The Simulation about Proposition 4

Reference


Thank You Very Much!